Journal of Structural Geology, Vol. 2, No. 1/2, pp. 149 to 158.
Pergamon Press Ltd. 1980. Printed in Great Britain.

0191-8141/80/0601-0149 $02.00/0

Strain heating and thermal softening in continental shear zones: a review

J. P. BRUN and P. R. COBBOLD

Centre Armoricain d'Etude Structurale des Socles
(CNRS - Université de Rennes) Campus de Beaulieu, 35042 Rennes Cedex, France

(*Received* 20 *June* 1979; *accepted in revised form* 19 *July* 1979)

Abstract—Strain heating results from the conversion of mechanical energy into heat during progressive deformation. As a physical phenomenon it is well known in fluid mechanics and has been studied theoretically and experimentally. This work has been extended by recent geophysical developments. Here we review (a) the fluid mechanics and geophysical work and (b) its application to continental shear zones.

The models indicate that temperature rises of a few hundred degrees can be expected in major shear zones (transcurrent shear zones, the bases of thrust sheets, or the margins of large diapirs). In certain special situations (some thrust sheets and nappes), even larger rises are possible. The resulting temperature gradients should be detectable geologically, but evidence is scanty. The resulting thermal softening is sufficient to concentrate most of the deformation in narrow zones. Thus strain heating is an important crustal phenomenon which should be incorporated in models of large-scale tectonic processes. It may even contribute to local partial melting in some shear zones.

INTRODUCTION

STRAIN heating results from the conversion of mechanical energy into heat during progressive deformation. As a physical phenomenon it is well known in fluid mechanics and has been studied theoretically and experimentally. This knowledge has been incorporated in recent geophysical developments. The purpose of this paper is to review (a) the fluid mechanics and geophysical work, and (b) the application of strain heating to geophysical and geological problems. We aim to show that strain heating is a significant phenomenon in the continental crust and that its effects should be (a) incorporated in all mechanical models of large-scale deformation, and (b) borne in mind by the geologist when he studies relic thermal gradients and deformation zones. We focus our attention on shear zones, because (i) deformation is strongly concentrated and relatively large amounts of energy are dissipated; and (ii) the boundary conditions impose physical constraints that lead to simplifications in the mathematical formulation of models.

Of the energy dissipated during permanent deformation, some may be converted into heat and some may be used to change the physical state of the deforming materials (for example, the mineral phases present, or their microstructural state). Most models of strain heating ignore the latter effect and assume that all the mechanical energy is converted into heat. Although we will here adopt the same assumption, it is important to bear in mind that geological deformation is often accompanied by changes in physical state (e.g. syntectonic metamorphism). In some situations, endothermic reactions may consume a significantly large proportion of the available energy.

Of the energy converted into heat, some is stored locally and leads to a rise in temperature, whereas the rest is removed, for example by thermal conduction in the solid state, or by circulating fluids. Here we will ignore the latter effect and make the special assumption that heat is transferred only in the solid state. Under these conditions we are considering the following energy balance:

$$\text{ENERGY STORED} = \text{MECHANICAL ENERGY DISSIPATED} + \text{ENERGY GAINED (OR LOST) BY THERMAL CONDUCTION} \quad (1)$$

A large part of this paper deals with solutions of (1) under boundary conditions appropriate to shear zones.

It is important to realize that (1) does not always describe a steady process. If the rate of energy dissipated exceeds the rate of heat loss by conduction, the temperature must rise. The rheology of most rocks is strongly temperature-dependent, and a rise in temperature causes a softening effect. Such thermal softening may allow an increased rate of deformation and thus an increased rate of energy dissipation. Further temperature rises may ensue, leading to further softening and so on. The general process is referred to as thermal feedback. Under certain circumstances, the temperature can increase so quickly that deformation is effectively unstable: this is referred to as thermal runaway. Under other circumstances, a steady state may be achieved where the energy produced mechanically is continuously balanced by that lost by conduction. Thus (1) is in general time-dependent.

The amount of thermal softening and its feedback effects are strongly dependent on the flow-laws chosen for the strain-heating models. We discuss these aspects first, concentrating on the temperature-dependence of the flow-laws. Next we discuss briefly how stresses and strain-rates may vary across a shear zone, as this provides important constraints on the models.

Models of strain heating are introduced and described in increasing order of complexity. Thus we discuss first the steady-state solutions for Newtonian materials with a simple temperature-dependence; then we introduce more realistic temperature-dependences and non-Newtonian behaviour; lastly we consider some aspects of time-dependent behaviour. This is not strictly the order in which developments appeared historically, for which we refer the interested reader to the excellent reviews by Sukanek & Laurence (1974) and Schubert & Yuen (1978).

In a section on geological applications, we consider some of the possible uses and misuses of the models. The subject is certainly in its infancy and the conclusions are limited by the lack of available data on rheological properties and boundary conditions for the systems considered. For these reasons the applications must be interpreted cautiously. Nevertheless we feel the subject may be fundamental to an understanding of tectonic processes, and we have thus attempted to clarify some of the principles involved.

RHEOLOGICAL PARAMETERS

Flow laws and thermal softening

Our knowledge and understanding of the rheology of the continental crust is far from complete but it is expanding. Here we will discuss some aspects relevant to models of strain heating.

For a rock with a given composition and structure, the flow characteristics depend heavily on three parameters, stress, temperature and strain-rate (Heard 1976). In general, two of these may be taken as independent variables. Measurements of strain in natural rocks and laboratory experiments indicate that large volume changes are rare. On the scale of the continental crust, the effects of mean stress (lithostatic pressure) are also not likely to be highly significant. Thus we may consider primarily the deviatoric components of both stress and strain-rate tensors.

All variables may conveniently be linked by means of constitutive equations (flow-laws). The exact form of such equations depends on the mechanism of deformation that is dominant (see Weertman & Weertman 1975). Under most geological conditions extant in the continental crust, flow is largely achieved by deformation mechanisms that are diffusion-controlled (Heard 1976). Amongst these are dislocation creep, Coble creep, Nabarro–Herring creep and pressure-solution (see Rutter 1976, McClay 1977). The corresponding law for steady-state flow in an isotropic rock contains a power-law dependence of strain-rate upon stress. It can be written (Nye 1953):

$$\dot{e}'_{ij} = (KD/RT)\Sigma'_2{}^{\frac{n-1}{2}}\sigma'_{ij} \qquad (2)$$

where $\dot{e}'_{ij}$ is the tensor of deviatoric strain-rate, $\sigma' ij$ that of deviatoric stress, Σ'_2 is the second invariant of the latter, T is the thermodynamic temperature, R is the gas constant, D is the diffusivity of the rate-controlling process, n is the stress-exponent, and K is a constant. Such an equation has been predicted theoretically and verified experimentally for polycrystalline monominerallic rocks, including calcite (Heard & Raleigh 1972, Rutter 1976, Schmid *et al.* 1977), quartz (Rutter 1976, Parrish *et al.* 1976) and olivine (Post 1977, Kohlstedt & Goetze 1974). Polymineralic rocks are likely to behave in the same way (Stocker & Ashby 1973, Weertman & Weertman 1975, Heard 1976).

In equation (2), the diffusivity D is temperature-dependent:

$$D = D_0 \exp(-H/R\,T) \qquad (3)$$

where D_0 is a constant, H is the activation enthalpy of the diffusion process and $\exp(-H/R\,T)$ is the Arrhenius factor. This factor is of great importance, as it accounts for most of the temperature dependence of creep. Its effects are most strongly felt over a certain temperature range that depends on the value of the activation enthalpy for the material in question. For most rocks, this value is within the range 0–300 kJ mol^{-1} (Heard 1976), and the effects of the Arrhenius factor are most strongly felt at temperatures below the melting point (Fig. 1). For example, in the range 300°C < T 1 800°C,

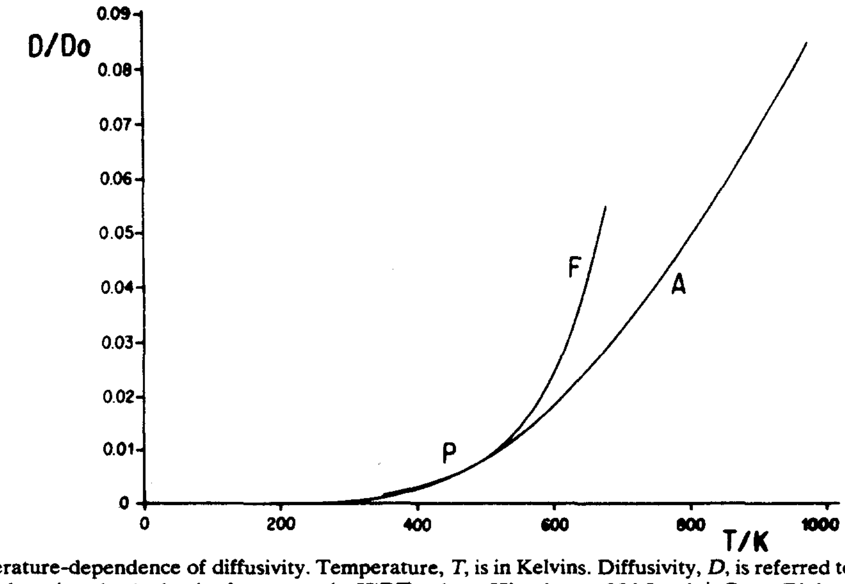

Fig. 1. Temperature-dependence of diffusivity. Temperature, T, is in Kelvins. Diffusivity, D, is referred to constant value, D_0. Curve A is based on the Arrhenius factor, $\exp(-H/RT)$, where H is taken as 20 kJ mol^{-1}. Curve F is based on the Frank-Kamenetzky approximation about point P.

and for an activation enthalpy of 200 kJ mol^{-1} (typical of calcite), a temperature increase of 100° at constant stress corresponds to a tenfold increase in strain-rate. It is perhaps surprising that such a dramatic effect has so often been neglected in the modelling of continental processes.

The Arrhenius factor contains an exponential dependence on reciprocal temperature which is mathematically inconvenient. Hence it is often replaced by a direct exponential dependence, exp (aT) (Frank-Kamenetzky 1939). The Frank-Kamenetzky approximation is obtained by retaining only the first two terms in a Taylor expansion of (H/RT) in the vicinity of some reference temperature, T_0: this yields $a = H/RT^2$. The approximation is good within the range $T_0 - 50° < T < T_0 + 50°$ (Fig. 1), but beyond this its use may lead to significant errors.

It should be pointed out that temperature is not necessarily the only softening agent in (2). Thus the stress exponent, n, also plays a part (see Poirier 1980). For many rocks, however, it is in the range $1 < n < 5$ (Heard 1976): this means that strain-rate is more sensitive to variations in temperature, than it is to variations in stress. Other softening factors, implicit in the parameter K of (2) are strain-softening, geometric or rotation softening (Cobbold 1977, Poirier 1980) and changes in structure (Poirier 1980). Very few quantitative data are available for these processes and they will not be considered in what follows. Structural changes associated with strain softening often lead to anisotropic behaviour, so that (2) may be oversimplified in this respect. Some of the consequences of anisotropic rheology in the development of shear zones have been outlined by Cobbold (1977): they will not be considered here.

For an isotropic rock undergoing simple shear such that the shear direction is the coordinate axis x, the plane of shear is xy and the velocity along x is u, equation (2) reduced to:

$$\dot{\gamma} = \frac{\partial u}{\partial y} = (2B/T) \exp(-H/RT)\sigma_s^n \qquad (4)$$

where $\sigma_s = \sigma'_{xy}$ is the shear stress, $\dot{\gamma} = 2\dot{e}_{xy}$ is the shear strain-rate and $B = KD_0/R$ is a constant. This is the equation that will be used in the strain heating models of the next section. It should perhaps be emphasized that the equation describes a steady state flow, yet strain heating leads in general to time-dependent flow. There is obviously a need for experimental work on rocks under conditions of variable temperature: until this is done the use of (4) can be criticized.

Strain-rate and stress distribution

We will consider idealized models of shear zones, with plane parallel margins (Ramsay & Graham 1970). The kinematics of such zones are simple: in the absence of (a) deformation outside the zones, and (b) volume changes, all internal deformation is accomplished by shearing parallel to the margins. The shear direction and the width of the zone are constant in space. We will also assume that the former is invariant with time. Thus we need consider only the shear strain-rate, $\dot{\gamma} = 2\dot{e}_{xy}$, which may vary across the shear zone.

To respect the equations of stress equilibrium, in the absence of gravitational forces, the shear stress and normal stress acting on the margins must also be constant across the entire zone (see e.g. Cobbold 1977). If the material is isotropic, we need consider only the shear stress (equation 4). If gravitational forces are significant, as in a thrust sheet or nappe, the shear stress increases with depth (see Elliott 1976). For constant density and surface slope, the increase is linear.

STRAIN HEATING MODELS

General equations

For an isotropic medium, the energy balance equation (1) can be written explicitly as (Carslaw & Jaeger 1959):

$$C\frac{\partial T}{\partial t} = k\nabla^2 T + \dot{E}. \qquad (5)$$

The term on the left-hand side represents the rate at which temperature increases with time, t. This is governed by the volumetric specific heat, c. On the right, the first term represents the rate of heat loss by conduction, this being governed by the thermal conductivity, K, and the temperature distribution $(\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2})$. The second term, $\dot{E}$, is the rate of energy dissipation per unit volume of material. It is given by:

$$\dot{E} = \sigma_{ij}\dot{e}_{ij} \qquad (6)$$

where σ_{ij} is the strain tensor and $\dot{e}_{ij}$ the strain-rate tensor. For simple shear, (6) takes the simple form:

$$\dot{E} = \sigma_s\dot{\gamma} = \sigma_s\frac{\partial u}{\partial y}. \qquad (7)$$

In (5), both c and k are generally taken to be constant, because for natural materials their temperature-dependence is generally small; also this assumption makes the equation easier to solve.

Another simplification is introduced if we consider only the ideal shear zone, for which all variations are one-dimensional, that is, normal to the boundaries. Taking the boundaries normal to y, (5) becomes:

$$c\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + \dot{E}. \qquad (8)$$

Use of this equation is probably justified for thrust sheets and nappes, where heat flow is mostly vertically directed. Justification is more difficult for transcurrent shear zones near the upper surface of the crust, as here a significant proportion of the heat flow is directed vertically and hence parallel to the boundaries of the zone.

It is convenient to consider three special limits of (8), for each of which one of the terms vanishes. If the first term vanishes, we have:

$$k\frac{\partial^2 T}{\partial y^2} = -\dot{E}. \qquad (9)$$

If $\dot{E}$ does not vary with time, (9) represents a steady state. Whether or not such a state is ever truly attained geologically, the study of (9) is a useful if not necessary preliminary to the study of time-dependent solutions. Much of the work reviewed in this paper concerns solutions of (9). If in (8) the second term vanishes, we have:

$$c\frac{\partial T}{\partial t} = \dot{E}. \qquad (10)$$

Here no heat is conducted throughout the material and the system is therefore at all points adiabatic. Such a situation never actually occurs, but it is useful as a first approximation for materials with very low thermal conductivity or for situations where thermal gradients are very small. Geologically, the use of (10) may be justified occasionally; but in general it is dangerous, because the rate of heat loss by conduction often matches or even surpasses the rate of heat production by deformation.

Finally if in (8) the third term vanishes, we have:

$$c\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2}. \qquad (11)$$

This corresponds to a zero heat source, and is the equation of heat conduction in one dimension. Solutions to this equation are well known (Carslaw & Jaeger 1959). They are useful models for what happens if internal deformation ceases.

Steady state solutions

Substitution of (7) in (9) gives:

$$k\frac{\partial^2 T}{\partial y^2} + \sigma_s\frac{\partial u}{\partial y} = 0. \qquad (12)$$

For a steady-state, the stress σ_s is constant in time. In the absence of body forces it is also constant in space and (12) can therefore be integrated, giving (Gavis & Laurence 1968, Yuen *et al.* 1978).

$$k\frac{dT}{dy} + \sigma_s u = 0. \qquad (13)$$

No further integration is possible without introducing the rheological properties. Substitution of the power-law (4) into (12) gives:

$$k\frac{d^2T}{dy^2} + (\sigma_s)^{n+1}\,(2B/T)\,\exp\,(-H/RT) = 0. \qquad (14)$$

This is a nonlinear differential equation of the second order in T. If we use the Frank-Kamenetzky approximation, $\exp(aT)$, instead of the term $(2B/T)\exp(-H/RT)$, analytical solutions of this equation may be found. A large amount of theoretical work concerning this approach is reviewed historically and discussed by Sukanek & Laurence (1974). For a known boundary temperature, T_0, Gavis & Laurence (1968a) give an expression for the temperature within a slab of constant thickness undergoing heterogeneous shear (plane Couette flow). Early solutions of (14) assume Newtonian behaviour (n = 1), which leads to simple analytical expressions for the shear stress and shear rate across the slab. It is convenient to express these by means of two-dimensionless parameters, the Brinkman number, Br, and the Gruntfest parameter, G (Gruntfest 1963). These are defined as

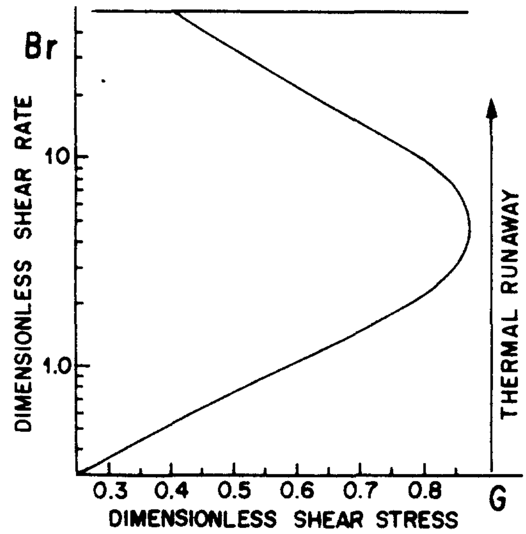

Fig. 2. Steady-state solution for a Newtonian slab, with Frank-Kamenetzky approximation. The Brinkman number, Br, is plotted against the Gruntfest parameter, G. Thermal runaway occurs if G is maintained at a value greater than 0.88.

$$\mathrm{Br} = \frac{a\mu_0 U^2}{k};\ \mathrm{G} = \frac{ah^2\sigma_s^2}{k\mu_0} = ah^2\dot{E}_0/k \qquad (15)$$

where μ_0 is the viscosity at temperature T_0, U is the boundary value of the velocity u, and h is the half-thickness of the slab. The solution of (14) is found to be single-valued in terms of Br but double-valued in terms of G (Joseph 1964, Gavis & Laurence 1968a). It can be written (Sukanek & Laurence 1974):

$$G = \frac{4}{(2 + \mathrm{Br})}\arcsin\left\{\frac{\mathrm{Br}^{\frac{1}{2}}}{2}\right\}. \qquad (16)$$

The parameter G has a maximum possible value G_L of 0.88 (Fig. 2), for which the Brinkman number has a value Br_L of 4.55. For $\mathrm{G} > \mathrm{G}_L$, no steady shear flow exists. For $\mathrm{G} < \mathrm{G}_L$, there are two possible values of the Brinkman number. Flows with $\mathrm{Br} < \mathrm{Br}_L$ have been termed subcritical (see Schubert & Yuen 1978), and those with $\mathrm{Br} > \mathrm{Br}_L$, supercritical. Analytical expressions also exist for the temperature and velocity profiles across the slab (Gavis & Laurence 1968). The temperature varies in an almost parabolic way, reaching a maximum in the centre of the slab (Fig. 3). In consequence the velocity also reaches a maximum in the centre.

The analytical solution (16) has been confirmed by numerical calculations (Gruntfest 1963) which have yielded the same maximum value of $\mathrm{G} = 0.88$ for a steady state. The physical reality of the solutions, and especially that of the supercritical branch, was for a long time open to question. It was finally confirmed by experiments on Newtonian fluids (Sukanek & Laurence 1974). These demonstrated the existence of a maximum shear stress ($\mathrm{G} \approx 0.88$) at a certain shear rate ($\mathrm{Br} \approx 4.5$). The behaviour can be explained as follows.

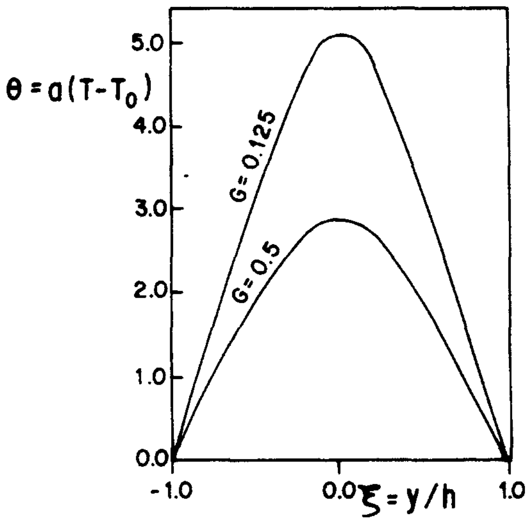

Fig. 3. Temperature distribution for steady-state shearing of Newtonian slab. Dimensionless temperature, $\theta = a\ (T - T_0)$, is plotted against dimensionless position, $\xi = y/h$, for two values of the Gruntfest parameter, G.

At low shear rates, the rate of shear strain increases monotonically with stress, according to the stress-exponent, n. The heat produced within the slab is negligible. At higher shear rates, shear heating is significant, but more than matched by the rate of heat conduction: thus the rate of shear strain still increases with stress, although thermal softening has an effect. Above the critical rate (Br = 4.55), heat is produced faster than it can be conducted out of the system: the material becomes softer and the stress drops. In general, for a given stress, there are two possible states, a cold one (subcritical branch) and a warm one (supercritical branch).

A similar behaviour is exhibited by a material obeying a power law as in (4) (Gavis & Laurence 1968b). This is to be expected, because the solution of (14) at constant stress gives the same temperature distribution for any value of n.

Solutions of (14), with the full Arrhenius factor instead of the Frank-Kamenetzky approximation, have been studied by Yuen & Schubert (1977), and Schubert & Yuen (1978), mainly in connection with models of shear flow in the asthenosphere. The general approach is reviewed by Yuen & Schubert (1979). They show that use of the Arrhenius factor leads to the solutions having a third branch (the hot branch), as well as the subcritical and supercritical branches. The hot branch exists for all high values of G (Fig. 4). It can be explained as follows. At high shear rates, strain heating causes large temperature rises. The Arrhenius factor then tends towards a limiting value (see section on flow laws). Hence the thermal factor becomes almost negligible and stress once again increases with strain rate, as it does on the cold subcritical branch. It should be noted that the large temperature rises needed to attain the hot branch will in many natural materials cause changes in deformation mechanisms or in mineral phases, thus invalidating the model. The hot branch can be considered as an upper theoretical limit for the model considered.

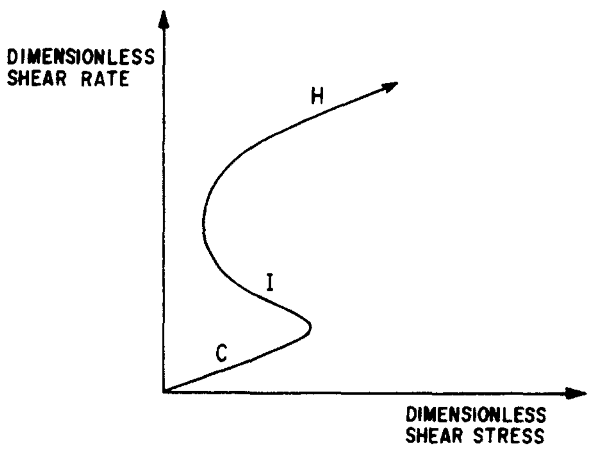

Fig. 4. Sketch of steady-state solution for Newtonian slab with Arrhenius factor. H—hot branch; I—intermediate or supercritical branch; C—cold or subcritical branch.

Using the full Arrhenius factor, Yuen *et al.* (1978) give an analytical solution for the maximum temperature, T_{max}, in the centre of a Newtonian slab. They show that:

$$U_0^2 = 4\ kB\ \{E_1\ (H/RT_{max}) - E_1\ (H/RT_0)\} \quad (17)$$

where u_0, T_0 are the velocity and temperature at the slab boundaries and E_1 is a special function, the exponential integral. It follows that for a given boundary velocity (to which corresponds a unique value of boundary stress), the maximum temperature in the centre of the slab is independent of the slab thickness. If $T_0 \ll T_{max}$ (17) is well approximated by the simpler expression:

$$U_0^2 = (16\ kBRT_{max}^2/H)\ \exp\ (-H/RT_{max}). \quad (18)$$

The accuracy of (18) has been verified by numerical models incorporating a full-time dependence (Yuen *et al.* 1978). Similar results have also been obtained by graphical integration (Turcotte & Oxburgh 1968).

The existence of three branches of the steady-state solutions has been confirmed numerically by Clarke *et al.* (1977) even for situations where there is a geothermal heat flux normal to a slab, and where the shear stress varies linearly with depth, as might be expected in glaciers and ice-sheets. Instead of the Gruntfest parameter, G, Clarke *et al.* (1977) use a stability parameter, β, defined as:

$$\beta = ah^2\dot{E}_b/k. \quad (19)$$

Here $\dot{E}_b$ is the rate of energy dissipation at the base of the slab, when $T = T_0$. The solutions (Fig. 5) can be presented in terms of three parameters: one is the stability parameter, β, another is a dimensionless geothermal parameter, φ; the third is the dimensionless surface temperature gradient. For high values of φ, the solution has only one branch, but for low values it has three branches.

If the shear stress and hence β are increased from some initially low value, there will be a positive jump in temperature and velocity once the critical value β_1 is exceeded (Fig. 5). Similarly if β decreases from some initially high value, there will be a negative jump near the other critical value β_0. The points are referred to in the mathematical literature as bifurcation points (Keller

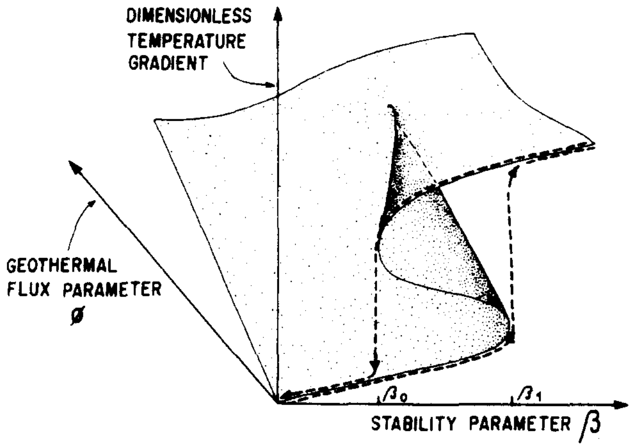

Fig. 5. Sketch of steady-state solution for ice-sheet or thrust-sheet.

& Antman 1969). Topologically, the surface (Fig. 5) corresponds to one of the seven elementary catastrophes known as a cusp, or Riemann-Hugoniot, catastrophe (Thom 1972). A positive temperature jump at β_1 would occur physically by a process of thermal runaway. Just how easily or at what point exactly this can occur must be investigated using either the full time-dependent equations, or else a more simplified stability analysis, known as a perturbation analysis. This will be discussed next.

Stability of the basic state

Much attention has been focussed on the stability of the steady-state solutions so far examined. Thus if we assume that a flow has a basic state which is steady, we may ask what happens following a small perturbation of the flow. Does the perturbation amplify significantly with time, so that the basic state is unstable; or does the perturbation decay, so that the flow returns to the basic state which is therefore stable?

It would appear important in stability investigations of this kind to distinguish between infinitesimal and finite perturbations. If perturbations are infinitesimal, the mathematical treatment of the problem becomes easier, because the equations to be solved are linearized. On the other hand, finite perturbations may have more of a destabilizing effect.

Using the one-dimensional time-dependent equation (8), Melosh (1976) argued that supercritical basic states are unstable in the presence of infinitesimal perturbations. The perturbation analysis assumed a condition of constant boundary stress. Yuen & Schubert (1977) and Schubert & Yuen (1978) have shown that the supercritical states are stable to infinitesimal perturbations if the surface velocity is maintained constant. All these analyses however assumed that the perturbations were one-dimensional.

Recently, Yuen & Schubert (1979) have investigated two-dimensional infinitesimal perturbations in materials with a power-law rheology. They have shown that all the basic states are stable to such perturbations. They also argue that three-dimensional perturbations should decay even more rapidly. However they do point out that thermal runaway could perhaps be induced by finite-amplitude disturbances, especially changes in stress.

Clarke *et al.* (1977) have considered the time constants for the growth of small perturbations superimposed on a subcritical basic state where stress increases linearly with depth. Specifically they have chosen perturbations due to increases in stress. Rapid growth of instability was obtained only for relatively large increases in stress. No analysis was performed for two-dimensional perturbations and, following Yuen & Schubert (1979), it is possible that the basic state is stable under these conditions.

Fully time-dependent solutions

Substitution of (4) and (7) into (8) gives the one-dimensional time dependent equation:

$$c\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + (\sigma_s)^{n+1}\,(2B/T)\exp(-H/RT). \quad (20)$$

If we use the Frank-Kamenetzky approximation, (20) becomes

$$c\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + (\sigma_s)^{n+1}\exp a\,(T - T_0). \quad (21)$$

This equation can be reduced to a dimensionless form by the substitutions

$$\theta = a\,(T - T_0);\ \xi = y/h;\ \tau = kt/ch^2. \quad (22)$$

We obtain:

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\xi^2} - \beta\exp\theta \quad (23)$$

where β is defined as in (19) and becomes the Gruntfest parameter, G, for Newtonian materials ($n = 1$). It was Gruntfest (1963) who showed numerically that solutions of (23) are critically dependent on the value of G (Fig. 6). For boundary conditions of constant stress and temperature a steady state is achieved provided $G < 0.88$. This is the critical value predicted analytically (see section on Steady-State Solutions). The time required to arrive at the steady state is approximately given by (Fig. 6):

$$\tau = 1. \quad (24)$$

For $G > 0.88$, no steady state is attained; instead the temperature increases superexponentially (Fig. 6). This is thermal runaway.

Griggs & Baker (1969) and Fujii & Uyeda (1974) also studied this behaviour under constant stress. Under constant strain-rates or constant boundary velocities, no thermal runaway is possible. Instead the system evolves until it reaches one of the branches of the steady state (Griggs & Baker 1969).

Using the full Arrhenius factor and a Newtonian rheology, Yuen *et al.* (1978) have studied the time-dependent evolution of an initial disturbance in an infinitely thick slab, with constant boundary velocity and temperature. If the disturbance is initially highly localized in a narrow zone within the slab, the tempera-

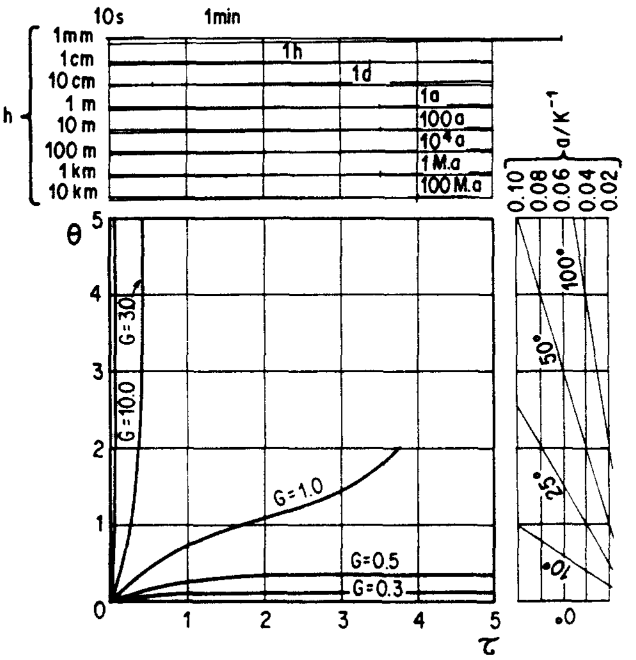

Fig. 6. Numerical results of Gruntfest (1963) for time-dependent model. Dimensionless temperature, θ, is plotted against dimensionless time, τ, for vårious values of Gruntfest parameter, G. True values of time are indicated (top) for various widths, *h*, of shear zone. True values of temperature (in degrees centigrade) are shown (right) for various values of the Frank-Kamenetzky exponent, *a*.

ture in that zone rapidly tends towards the maximum possible value predicted by the steady-state solutions, whilst outward conduction of heat causes the zone to widen slowly. Situations where the initial disturbance is not so highly concentrated initially are considered by Fleitout & Froidevaux (1980).

USES AND MISUSES OF MODELS

Geologists and geophysicists have long been aware that deformation is a potential source of thermal energy and this has led to many qualitative speculations concerning the relationships of deformation and metamorphism in orogenic belts. In this review we will consider only quantitative analyses. They may be listed in historical order; but we prefer instead to group them according to the number of parameters considered and the resulting complexity of the equations. We do not pretend that the review is complete.

Adiabatic models

These correspond to solutions of equation (10). Such models have the advantage of being relatively simple and of providing an upper limit on the temperature rises that can occur. It should not be forgotten that, by neglecting heat loss by conduction, they may dangerously overestimate the true temperature rise. The extent of such a rise depends on whether or not the models consider feedback effects and on the boundary conditions adopted.

Goguel (1948) was one of the first to calculate the amounts of energy dissipated in orogenic belts and the temperature rises this could produce. He did not consider feedback and was forced to conclude that deformation did not produce enough heat to account for regional metamorphism. More recently, Poirier *et al.* (1979) have proposed a model of shear zone development with thermal feedback under boundary conditions of constant strain-rate. They conclude that strain heating does not lead to significant temperature rises and that the resulting shear zone geometry is not very different from that produced under isothermal conditions. This result is somewhat surprising, but probably is an outcome of the assumption of constant strain-rate.

External heat source only

These models use the equations of heat conduction in one (equation 11) or more dimensions. Strictly they are not models of strain heating, but we mention them, because they have contributed strongly to geologists' understanding and recognition of thermal problems. Thus Graham & England (1976) considered frictionally generated heat on fault surfaces and determined the thermal history of adjacent rocks. Close correspondence was found with temperature deduced from metamorphic mineral assemblages.

Constant internal heat source

These models are based on equation (8) where $\dot{E}$ is assumed to be constant. Thus Reitan (1969) suggested that heat could be generated by frictional contact between the constituent grains of a rock and that this could lead to metamorphism. Unfortunately the assumption of energy dissipation constant in space requires that deformation be homogeneous: otherwise the equations of stress equilibrium are violated. This limits the usefulness of the model. The same criticism can be levelled at the model of shear zone development proposed by Nicolas *et al.* (1977). We conclude that these models are inappropriate for shear zones.

Thermal runaway under constant stress

The work of Gruntfest (1963) on solutions of equation (23) for Newtonian materials stimulated much interest in the possibility of thermal runaway occurring during natural processes. The advantage of this approach is that it is a simple matter to calculate the stability parameter G of (15) and so decide whether or not runaway will occur. The disadvantage is that it is necessary to know the width of the zone of shearing. Also the process cannot occur unless the applied shear stress remains constant or nearly so.

The general approach has been applied to problems of: (a) the genesis and emplacement of magmas (Shaw 1969, Fujii & Uyeda 1974, Hardee & Larson 1977); (b) thermal instability in the asthenosphere (Anderson & Perkins 1974, Melosh 1976, Schubert & Yuen 1978, Yuen & Schubert 1979); (c) the stability of ice-sheets

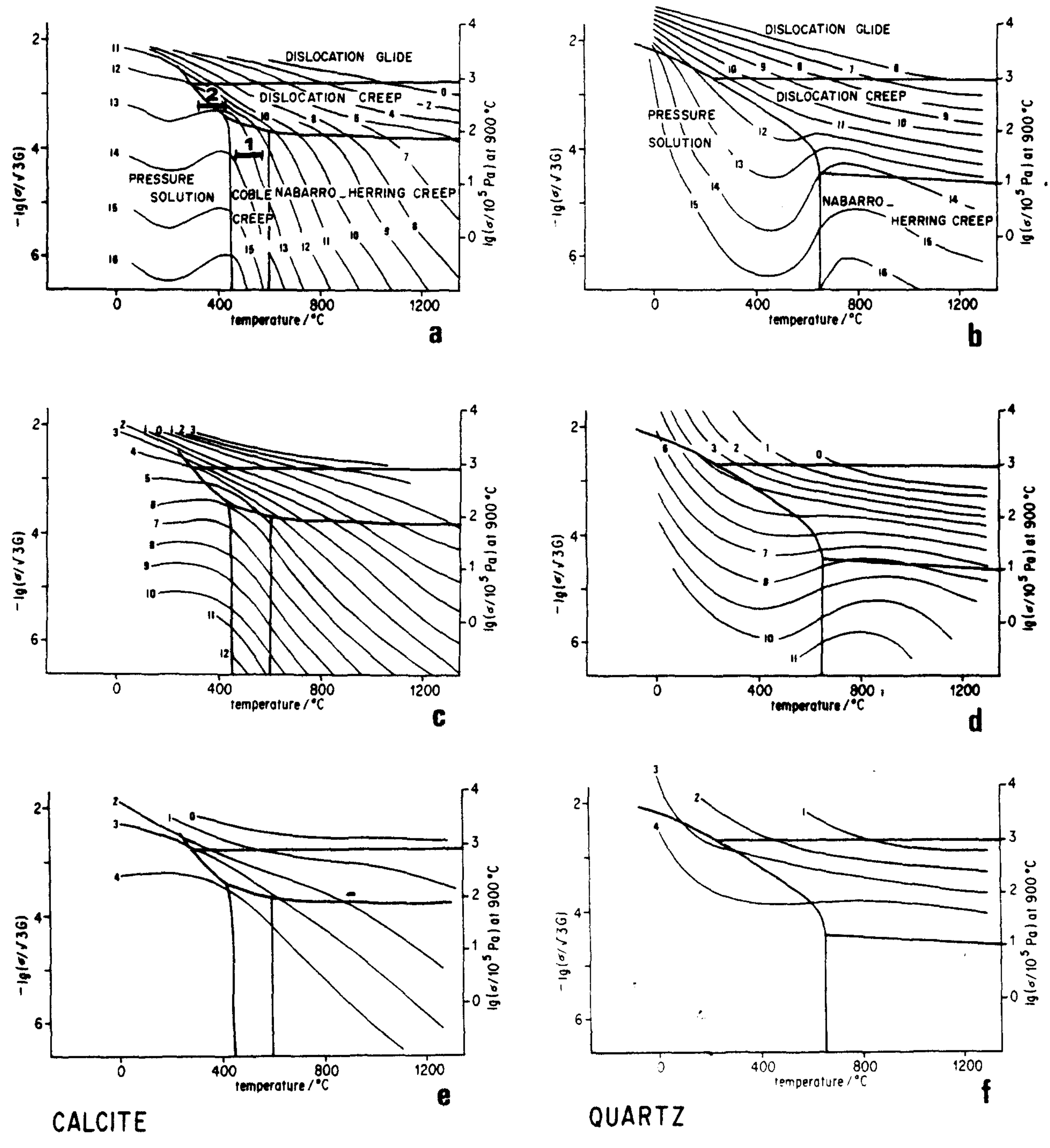

Fig. 7. Calculation of critical shear-zone widths for calcite (left) and quartz (right). Deformation mechanism maps (a & b) are taken from Rutter (1976). Differential stress, σ, normalized by the elastic modulus, G, is plotted against temperature. Curves are labelled for $-\log\{\dot{e}/s^{-1}\}$, where $\dot{e}$ is axial strain-rate in uniaxial compression. In (c) and (d), curves are labelled for $-\log\{\dot{E}/\mathrm{Wm}^{-3}\}$, where $\dot{E}$ is the rate of energy dissipation in simple shear. In (e) and (f), curves are labelled for $-\log\{h_c/m\}$, where h_c is the critical shear zone width.

(Clarke *et al.* 1977); and (d) shear zones in the continental crust (Cobbold & Brun 1977).

It is instructive to calculate the critical width h_c of a shear zone which must be exceeded for thermal runaway to occur under given conditions of stress and rheology. From (15) we have:

$$h_c = (Gk/a\dot{E}_0)^{\frac{1}{2}}. \qquad (25)$$

If we take a rock for which the mechanisms of deformation and strain-rates are known under various conditions of temperature and differential stress (Figs. 7a & b), it is possible to calculate the corresponding rates of energy dissipation (Figs. 7c & d) and the exponential constant, of the Frank-Kamenetzky approximation. Using a value of $k = 2.5\ \mathrm{Wm}^{-1}\ \mathrm{K}^{-1}$, and $G = 0.88$ (the critical value for onset of thermal runaway) we have calculated, by means of (25) the critical shear zone widths for thermal runaway in polycrystalline quartz or calcite (Figs. 7e & f). The results using this model indicate that thermal runaway is only possible for very wide zones (1 km or more) or very high differential stresses (greater than 1 kbar = 10^8 Pa). These conditions are difficult to meet in natural shear zones: so is the condition of steady applied stress, which is necessary for thermal runaway. One possibility which comes to mind is a thrust sheet or nappe, which is analogous to an ice sheet or glacier. For thrust sheets, the total thickness may easily exceed 5 km; but the basal shear stress is unlikely to exceed 50 bars (5 MPa) (Elliott 1976). These conditions are apparently not sufficient for the onset of runaway (Fig. 7), but they are close enough to warrant a better analysis. Thus we may consider the model of Clarke *et al.* (1977), originally proposed for ice-sheets, where the shear stress increases linearly with depth. Unfortunately, results are not available for the range of activation energies typical of many rocks: thus this subject remains to be investigated.

For runaway to occur in narrow zones (< 1 m), differential stresses must exceed 1 kbar (10^8 Pa). Such conditions may lead to seismic failure, thus rendering the ductile models inapplicable.

A factor not considered in analyses of thermal runaway is, of course, the time. Is enough time available for thermal runaway to become established? Are the displacements involved not too great? For problems of stability in the asthenosphere, Yuen & Schubert (1979) have applied equation (24) and decided that there may be just enough time to develop thermal runaway. For nappes and thrust sheets, where deformation is often concentrated in the lowermost zones and the total displacement may be in the order of tens of kilometres, there would appear to be no problem here either. Nevertheless, the subject remains open.

Steady state

The steady-state solutions discussed earlier may be used as models for steady geological situations where stress is not high enough for thermal runaway to occur. Thus Yuen *et al.* (1978), using equation (18), have calculated the maximum temperature T_{max} attained in the centre of a transcurrent shear zone subjected to a constant boundary velocity of 10 cm a^{-1}. The values obtained are independent of the shear zones thickness and of the boundary temperature provided this is not too high. In contrast, values depend on rheology: they are 1190, 999, 963, 834 and 619 K, for dry olivine, wet olivine, diabase, wet quartzite and limestone, respectively. The same values were also obtained by numerical methods. Such temperatures are surprisingly high; but then 10 cm a^{-1} is geologically a high velocity (spread over a width of 30 km, it corresponds to a mean strain-rate of about $10^{-13}s^{-1}$). The temperature gradient across the steady-state model implies that strain-rates in the centre are several orders of magnitude greater than at the margins. Thus most of the deformation is concentrated in the centre, and thermal softening, even in steady state, is a very effective localizing agent.

Full time-dependence

Fully time-dependent solutions of (20) have been applied by Yuen *et al.* (1978) to the initiation of a transcurrent shear zone in viscous crustal materials (see earlier sections). The approach provides interesting information on the thermal history at each point and on the widening of the zone of shear. The maximum temperatures attained are well predicted by the steady state solutions.

DISCUSSION

We wish to discuss certain points that we feel may be of interest to structural geologists, as well as suggest how further progress might be made.

Possibility of inducing partial melting

It has been suggested many times that shear heating could raise temperatures sufficiently to cause partial melting (e.g. Nicolas *et al.* 1977), leading in turn to diapiric uprise of granitic melts. This would explain certain associations of granitic bodies and shear zones in the continental crust. Conversely, it has been suggested (Poirier *et al.* 1979) that thermal softening produced by ascending plutons would conveniently localize a major shear zone, and explain the association more readily. The subject is of great interest in tectonics, but field evidence (in particular, geochronological data) is lacking.

Of the models discussed above, none of the physically valid ones predict temperatures high enough to yield partial melting, with the exception of those involving thermal runaway. But we have seen that thermal runaway is unlikely to occur in the crust except in seismic situations (narrow zones and high stresses) and possibly in some nappes and thrust-sheets: therefore it is tempting to conclude that shear heating cannot produce partial melting.

The conclusion is perhaps premature, because the models considered are still very simple. Thus they all assume, for example, that the material in the shear zone is compositionally homogeneous. Fleitout & Froidevaux (1980) have suggested that localized partial melting may occur in softer layers within a stratified sequence.

Rheological consequences of strain heating

Even if strain heating does not always have such spectacular consequences as partial melting, it must nevertheless exert a very significant control on deformation in large-scale shear zones. For example, at constant stress, even a rise in temperature of 100 K can lead to an order-of-magnitude increase in strain rate (Fig. 7a, 1). It may also in certain circumstances cause a change in deformation mechanism (Fig. 7a, 2). The steady-state models have no difficulty in predicting temperature rises of 100 K and this is sufficient to localize 90% of the deformation in the hotter zone. Such a mechanical effect cannot be neglected in any simulation of large-scale tectonic processes.

Consequences for the field geologist

All strain heating models for shear zones predict temperature differences and gradients which should be detectable in rocks. Evidence of such gradients (e.g. Nicolas *et al.* 1977) is so far scanty and controversial. Isotope· methods, detailed studies of deformation mechanisms, and studies of fluid inclusions or temperature-sensitive mineral parageneses, may eventually indicate whether the models are correct. In particular it may become possible to decide whether heat is consumed by other processes, such as circulation of fluids or endothermic mineral reactions.

Further prospects

The theoretical models are as yet too simple. More complex models and equations should perhaps be studied. Two-dimensional and three-dimensional models are already feasible, using numerical methods. Coupling of thermal and mechanical effects is perhaps best accomplished by finite element methods.

Acknowledgements—We are grateful to L. Fleitout for helpful discussions and for constructive criticism of the manuscript.

REFERENCES

Anderson, O. L. & Perkins, P. C. 1974. Runaway temperatures in the asthenosphere resulting from viscous heating. *J. geophys. Res.* **79**, 2136–2138.

Carslaw, H. S. & Jaeger, J. C. 1959. *Conduction of Heat in Solids.* Oxford University Press, Oxford.

Clarke, G. K. C., Nitsan, U. & Paterson, W. S. B. 1977. Strain heating and creep instability in glaciers and ice sheets. *Rev. Geophys. & Space Phys.* **15**, 235–247.

Cobbold, P. R. 1977. Description and origin of banded deformation structures. II Rheology and the growth of banded perturbations. *Can. J. Earth Sci.* **14**, 2510–2523.

Cobbold, P. R. & Brun, J. P. 1977. Strain heating and the development of crustal structures. (Abstract) Tectonic Studies Group Annual Meeting, Leeds.

Elliott, D. 1976. The motion of thrust sheets. *J. geophys. Res.* **81**, 949–963.

Fleitout, L. & Froidevaux, C. 1980. Thermal and mechanical evolution of shear zones. *J. Struct. Geol.* **2**, 159–164.

Frank-Kamenetzky, D. A. 1939. Calculation of thermal explosion limits. *Acta phys.-chin. URSS* **10**, 365–370.

Fujii, N. & Uyeda, S. 1974. Thermal instabilities during flow of magma in volcanic conduits. *J. geophys. Res.* **79**, 3367–3369.

Gavis, J. & Laurence, R. L. 1968a. Viscous heating in plane and circular flow between moving surfaces. *Ind. & Eng. Chem., Fundam.* **7**, 232–239.

Gavis, J. & Laurence, R. L. 1968b. Viscous heating of a power law liquid in plane flow. *Ind. & Eng. Chem., Fundam.* **7**, 525–527.

Goguel, J. 1948. Introduction à l'étude mécanique des déformations de l'écorce terrestre. *Mém. Serv. Carte géol. dét. Fr.* Imprimerie Nationale, Paris, 1–530.

Graham, C. M. & England, P. C. 1976. Thermal regimes and regional metamorphism in the vicinity of overthrust fault: an example of shear heating and inverted metamorphic zonation from Southern California. *Earth Planet. Sci. Lett.* **31**, 142–152.

Griggs, D. T. & Baker, D. W. 1969. The origin of deep-focus earthquakes. In: *Properties of Matter under Unusual Conditions* (edited by Mark, H. & Fernbach, S.) Interscience, New York, 23–42.

Gruntfest, I. J. 1963. Thermal feedback in liquid flow; plane shear at constant stress. *Trans. Soc. Rheol.* **7**, 195–207.

Hardee, H. C. & Larson, D. W. 1977. Viscous dissipation effects in magma conduits. *J. Volcanol. & Geotherm. Res.* **2**, 299–308.

Heard, H. C. 1976. Comparison of the flow properties of rocks at crustal conditions. *Phil. Trans. R. Soc.* **A283**, 173–816.

Heard, H. C. & Raleigh, C. B. 1972. Steady state flow in marble at 500°C–800°C. *Bull. geol. Soc. Am.* **83**, 935–956.

Joseph, D. D. 1964. Variable viscosity effects on the flow and stability of flow in channels and pipes. *Physics Fluids* **7**, 1761–1795.

Keller, J. B. & Antman, S. 1969. *Bifurcation Theory and Non Linear Eigen Value Problems.* W. A. Benjamin, New York.

Kohlstedt, D. L. & Goetze, C. 1974. Low-stress high-temperature creep in olivine single crystals. *J. geophys. Res.* **79**, 2045–2051.

McClay, K. R. 1977. Pressure solution and Coble creep in rocks and minerals: a review. *J. geol. Soc. Lond.* **134**, 57–70.

Melosh, H. J. 1976. Plate motion and thermal instability in the asthenosphere. *Tectonophysics* **35**, 363–390.

Nicolas, A., Bouchez, J. L., Blaise, J., Poirier, J. P. 1977. Geological aspects of deformation in continental shear zones. *Tectonophysics* **42**, 55–73.

Nye, J. F. 1953. The flow of ice from measurements in glacier tunnels, laboratory experiments and the Jungfraufirn bore hole experiment. *Proc. R. Soc.* **A219**, 477–489.

Parrish, D. K., Krivz, A. L. & Carter, N. L. 1976. Finite-element folds of similar geometry. *Tectonophysics* **32**, 183–207.

Poirier, J. P. 1980. Shear localization and shear instability in materials in the ductile field. *J. Struct. Geol.* **2**, 135–142.

Poirier, J. P., Bouchez, J. L. & Jonas, J. J. 1979. A dynamic model for aseismic ductile shear zones. *Earth Planet. Sci. Lett.* **45**, 441–453.

Post, R. L. 1977. High temperature creep of Mt. Burnett dunite. *Tectonophysics* **42**, 75–110.

Ramsay, J. G. & Graham, R. H. 1970. Strain-variation in shear-belts. *Can. J. Earth Sci.* **7**, 786–813.

Reitan, P. H. 1969. Temperatures with depth resulting from frictionally generated heat during metamorphism. *Mem. geol. Soc. Am.* **115**, 495–512.

Rutter, E. H. 1976. The kinetics of rock deformation by pressure solution. *Phil. Trans. R. Soc.* **A283**, 203–219.

Schmid, S. M., Boland, J. N. & Paterson, M. S. 1977. Superplastic flow in finegrained limestone. *Tectonophysics* **43**, 257–291.

Schubert, G. & Yuen, D. A. 1978. Shear heating instability in the earth's upper mantle. *Tectonophysics* **50**, 197–205.

Shaw, H. L. 1969. Rheology of basalt in the melting range. *J. Petrol.* **10**, 510–535.

Stocker, R. L. & Ashby, M. F. 1973. On the rheology of the upper mantle. *Rev. Geophys. & Space Phys.* **11**, 391–426.

Sukanek, P. C. & Laurence, R. L. 1974. An experimental investigation of viscous heating in some simple shear flows. *A. I. Ch. E. Jl* **20**, 474–484.

Thom, R. 1972. *Stabilité Structurelle et Morphogenèse.* W. A. Benjamin, New York.

Turcotte, D. L. & Oxburgh, E. R. 1968. A fluid theory for the deep structure of dip-slip fault zones. *Phys. Earth & Planet. Interiors* **1**, 381–386.

Weertman, J. & Weertman, J. R. 1975. High temperature creep of rock and mantle viscosity. *Annu. Rev. Earth Planet. Sci.* **3**, 293–315.

Yuen, D. A. & Schubert, G., 1977. Asthenospheric shear flow: thermally stable or unstable. *Geophys. Res. Lett.* **4**, 503-506.

Yuen, D. A., Fleitout, L., Schubert, G. & Froidevaux, C. 1978. Shear deformation zones along major transform faults and subducting slabs. *Geophys. J. R. astr. Soc.* **54**, 93–119.

Yuen, D. A. & Schubert, G. 1979. On the stability of frictionally heated shear flows in the asthenosphere. *Geophys. J. R. astr. Soc.* **57**, 189–207.